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Endogenous Growth, Backstop Technology Adoption and Optimal Jumps

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Abstract

We study a two-phase endogenous growth model in which the adoption of a backstop technology (e.g. solar) yields a sustained supply of essential energy inputs previously obtained from exhaustible resources (e.g. oil). Growth is knowledge-driven and the optimal timing of technology switching is determined by welfare maximization. The optimal path exhibits discrete jumps in endogenous variables: technology switching implies sudden reductions in consumption and output, an increase in the growth rate, and instantaneous adjustments in saving rates. Due to the positive growth effect, it is optimal to implement the new technology when its current consumption benefits are substantially lower than those generated by old technologies.

JEL codes O33, Q32, Q43.

Keywords Backstop technology, Discrete jumps, Endogenous growth, Exhaustible resources, Optimal Control.

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1 Introduction

One of the major challenges facing modern economies is the sustainability problem induced by resource dependence. Despite the rapid development guaranteed by technical progress, the production process of post-industrial economies still relies on a finite supply of minerals and fossil fuels, and the question of how to preserve individual welfare in the future is a worldwide political concern. In the last decade, a substantial body of economic literature tackled the issue of sustainability from the perspective of modern growth theory. Several authors analyzed the conditions under which technological progress is able to guarantee a sustained flow of output when exhaustible resources - e.g. oil - are essential inputs in production. Following the main insights of Stiglitz (1974), these contributions reformulated the problem in the context of endogenous growth models, where the conditions for achieving positive growth rates in the long run are intimately linked to the development of innovations and the profitability of R&D investment (Barbier, 1999; Sholz and Ziemes, 1999; Bretschger and Smulders, 2003). In this framework, the allocation mechanism is derived from intertemporal utility maximization *à la* Ramsey, and a crucial sustainability condition is that the rate of resource-augmenting technical progress be sufficiently high relative to the utility discount rate (Di Maria and Valente, 2008).[1]

This strand of literature addresses the issue of resource substitution only to some extent. Endogenous growth models exhibit long-run equilibria where production possibilities are sustained by the accumulation of knowledge-type capital. This form of technical progress progressively substitutes the resource in the sense that the increased efficiency of knowledge capital compensates, in terms of productivity, the restrictions imposed by resource scarcity on production possibilities. Since this mechanism does not make the resource 'superfluous' in finite time, the transitional dynamics of consumption and output are smooth. It may be argued that the substitution process is quite different when perfect substitutes of the resource exist: if the availability of new inputs makes the resource-based technology obsolete, the traditional method of production is abandoned in finite time, and the transition to resource-free techniques may involve non-smooth transitional dynamics. However, this issue has not been addressed in endogenous growth models, and this is a main motivation for this paper.

At the theoretical level, the analysis of technology adoption in models with exhaustible resources was pioneered by Hoel (1978), Dasgupta and Stiglitz (1981), and Dasgupta et al. (1982). In this framework, resource scarcity sets limits to economic activity in the long run, and the production process can be perpetuated only by implementing a *backstop technology* - i.e. a new method of production whereby exhaustible natural inputs are replaced by alternative, non-scarce factors (Nordhaus et al. 1973). The early literature treats the flow of extracted resources as a normal consumption good entering the intertemporal utility function. In this case, the resource demand schedule is determined by the marginal willingness to pay, and is stable over time. This implies that the criterion for backstop technology adoption is essentially *price-based*. Agents compare the time profiles of the costs of alternative sources of benefits - say, oil versus solar energy - and decide how much and for how long the exhaustible resource should be consumed. Implementing this reasoning in a model of optimal extraction, Hoel (1978) showed that energy prices are increasing in the short run, and then stabilized by backstop technologies in the long run. The reason is that, in the short run, the economy is resource-based and energy prices follow the behavior of net resource rents, that grow over time at a rate equal to the interest rate by Hotelling's rule. When resource-based energy becomes as expensive as solar-based energy, the latter method is adopted, and the non-scarce nature of solar inputs stabilizes energy prices from that point onward. The subsequent literature extended this model to include different market structures (Dasgupta et al. 1982), uncertainty (Dasgupta and Stiglitz, 1981; Hung and Quyen, 1993), and pollution (Tahvonen, 1997). While these additional features affect the optimal timing of technology adoption, the underlying criterion is substantially unaltered: when resource rents achieve a critical threshold set by the marginal reward of solar energy, the transition to solar-based energy is complete.

The above discussion clarifies that, although backstop technology adoption is a crucial issue for analyzing the sustainability problem, resource-based growth and substitute technologies are usually studied within different frameworks. Merging the two approaches is likely to generate substantial differences in the results for two independent reasons. First, endogenous growth models treat natural resources as an input that is combined with other man-made production factors. Since economic development is driven by investment rates, the resource demand schedule is not stable over time, and the time profile of resource use is crucially affected by the accumulation of the productive stocks representing the engine of growth. In this context, the adoption of a backstop technology would generate a combination of growth effects and level effects: as the economy switches to the new technology, the growth rate is modified because saving behavior changes depending on whether exhaustible resources are used in production or not. These growth effects matter for the optimal timing of technology adoption, but are generally neglected in the early literature. A second source of differences is that the optimality criterion employed in growth models is that of consumption-utility maximization with intertemporal discounting. If the timing of backstop technology adoption is chosen according to this criterion of present-value optimality, technology switching is determined by a *welfare-based approach*. Building on these points, this paper studies how endogenous growth models currently used in the sustainability literature can provide a more complete criterion for optimizing the timing of backstop technology adoption.

We assume that aggregate production requires energy, initially produced by means of exhaustible resources like oil. The backstop technology is represented by solar-based energy, and a benevolent social planner decides whether and when to abandon traditional oil-based energy in favor of the new technology. The optimal switching time is derived by applying optimal control theory, and the main results are as follows. The optimal path is characterized by a trade-off between the positive level effects produced by the resource-based technology during the first phase, and the growth-enhancing effects generated by the solar-based technology in the second phase. There always exists an interior solution to the optimal timing problem, characterized by discrete jumps in consumption and output. The adoption of the backstop technology implies a sudden reduction in consumption and output levels, a sudden increase in the growth rate, as well as instantaneous adjustments in consumption/saving propensities. The marginal productivities of the competing energy sources evaluated at the instant of technology switching may differ substantially. Due to the contrasting effects of technology switching on levels and growth rates, the adoption of new solar-based techniques is optimal even though the associated current benefits in terms of consumption (evaluated at the optimal switching instant) are substantially lower than those generated by the traditional resource-based technology.

The plan of the paper is as follows. Section 2 describes the main assumptions of the model and specifies the social problem in terms of a a two-phase optimal control problem. Section 3 analyzes the optimality conditions, characterizes the behavior of the economy in the two phases, and derives an explicit expression for the optimal switching time under isoelastic preferences. Section 4 completes the characterization of the optimal path, and derives the main results regarding the economic consequences of backstop technology adoption. Section 5 briefly discusses the connections with previous literature, and section 6 concludes.

2 Endogenous Growth with Backstop Technology

2.1 Assumptions

The general scheme is as follows. Time is continuous and indexed by $t \in [0, \infty)$. Before instant $t = 0$, the economy is resource-based: aggregate production is obtained by means of labor and energy inputs that consist of non-renewable resources - e.g. oil - extracted from a finite stock. At time $t = 0$ a new technology is available: energy can be obtained by means of a different method of production whereby exhaustible resources are replaced by a non-scarce input - e.g. solar energy. A benevolent social planner, endowed with perfect foresight and full control over allocations, decides whether and when to adopt the solar-based technology. The transition from resource-based to solar-based technologies is irreversible, and may take place at any instant from time zero onwards. We denote by $T \in [0, \infty)$ the instant in which this structural change takes place. The possibility of delaying the adoption of the backstop technology after time $t = 0$ implies that the economy will generally experience two different phases over the interval $t \in [0, \infty)$. During 'phase 1', delimited by $t \in [0, T)$, the economy is still resource-dependent. During 'phase 2', delimited by $t \in (T, \infty)$, the economy is solar-based.

The reference framework for modelling economic dynamics is provided by endogenous growth theories. In order to keep the analysis tractable, we will use a fairly simple model of balanced growth. Aggregate output is represented by $Y_i = AF_i(E_i, N)$, where A is the state of technology determined by the current stock of knowledge, E is energy, N is labor, and $i = 1, 2$ is the phase index. Output can be either consumed or invested in knowledge-improving activities - e.g. R&D activity - that enhance future production possibilities. The investment rate determines the growth rate of knowledge, $\dot{A}/A$, which is essentially the Hicks-neutral rate of technological progress in the economy. Assuming decreasing marginal returns to energy and labor, this general scheme can be rationalized in terms of several models where the role of the knowledge stock is played by different engines of growth - e.g. human capital accumulation (Lucas, 1988), learning-by-doing (Romer, 1989), or expanding varieties of intermediate inputs (Barro and Sala-i-Martin, 2004). In the present context, we assume that the same kind of knowledge $A(t)$ is exploited in both phases though it may display different productivity levels. The underlying reasoning is that knowledge productivity varies depending on whether it is applied to a solar-based or a resource-based production process. Denoting aggregate output in the two phases by Y_1 and Y_2, respectively, the technologies read

$$Y_1(t) = A(t) \cdot F(nR(t), N) = A(t) \cdot (nR(t))^{\delta} N^{1-\delta}, \tag{1}$$
$$Y_2(t) = \alpha A(t) \cdot F(mG, N) = \alpha A(t) \cdot (mG)^{\gamma} N^{1-\gamma}, \tag{2}$$

where labor N is fixed and inelastically supplied, $R(t)$ is the amount of resources extracted at time t from a finite resource stock, G is the constant flow of solar energy units available in each instant, and n and m are constant coefficients yielding energy-equivalent measures for the flows of resource and solar units, respectively. Parameter $\alpha > 0$ determines whether knowledge A is more productive ($\alpha > 1$) or less productive ($\alpha < 1$) in the second phase with respect to the first phase. The productivity parameters δ and γ lie between zero and unity, and are generally different as the production elasticity of exhaustible resources is not necessarily equal to the production elasticity of solar-based energy.

Technology (1) is exploited in the interval $t \in [0, T)$, whereas technology (2) is used from time T to infinity. In both phases, the aggregate constraint of the economy reads $Y_i(t) = C_i(t) + D_i(t)$, where C_i is consumption and D_i is investment in knowledge-improving activities in phase $i = 1, 2$. The aggregate constraint can be imposed by means of the relation

$$c_i(t) = 1 - d_i(t), \qquad i = 1, 2, \tag{3}$$

where the propensity to consume $c_i \equiv C_i/Y_i$ equals one minus the investment rate $d_i \equiv D_i/Y_i$. The engine of growth in each phase is knowledge accumulation. In general, production possibilities are enhanced by virtue of accumulation laws of the type

$$\dot{A}(t) = \varphi\left(A(t)^{+}, d_i(t)^{+}\right), \qquad i = 1, 2,$$

where $\partial\varphi/\partial A > 0$ represents a knowledge-stock effect that is conceptually equivalent to assuming e.g. increasing returns to human capital accumulation (Lucas, 1988), spillovers from past R&D activity (Aghion and Howitt, 1998) or, more generally, knowledge spillovers (Acemoglu, 2002). Assumption $\partial\varphi/\partial d_i > 0$ implies that the accumulation of knowledge increases with the economy's rate of investment, consistently with standard models of balanced growth with endogenous R&D expenditures (Grossman and Helpman, 1991; Aghion and Howitt, 1998). In the present context, we will implement the linear specification

$$\dot{A}(t) = \psi A(t) d_i(t), \tag{4}$$

where $\psi > 0$ is a constant proportionality factor. The linear form (4) can be justified in several ways,[2] and is particularly useful in the present context as it allows us to obtain optimal balanced-growth paths involving no transitional dynamics - as e.g. in Rebelo (1991), Rivera-Batiz and Romer (1991), Barro and Sala-i-Martin (2004: Ch.6). Expression (4) allows for differences in the growth rates of knowledge between the two phases, as the use of different technologies generally implies different saving propensities, $d_1 \neq d_2$.

In the first phase, the resource-based technology (1) operates under the constraint imposed by the scarcity of the exhaustible resource. Denoting by S the resource stock, the instantaneous reduction in the 'natural capital' of the economy equals the rate of resource use:

$$-\dot{S}(t) = R(t). \tag{5}$$

Given the above assumptions, the social problem can be specified according to the standard welfare criterion of present-value maximization. In order to obtain explicit solutions, we assume that instantaneous utility takes the isoelastic form

$$U(C(t)) = \frac{C(t)^{1-\sigma} - 1}{1-\sigma}, \tag{6}$$

where $\sigma > 0$ is the inverse of the elasticity of intertemporal substitution in consumption. Letting $\sigma \to 1$, we obtain logarithmic preferences.

2.2 The social problem

The analysis focuses on optimal paths determined by the solution of a centralized social problem. The objective is to maximize the present discounted value of the stream of consumption benefits

$$V = \int_0^\infty U(C(t))\, e^{-\rho t} dt, \tag{7}$$

where $\rho > 0$ is the social discount rate. The possibility of switching between technology (1) and technology (2) requires re-formulating the optimization as a *two-stage problem*. In fact, our model falls in the class of optimal control problems with endogenous switching time studied e.g. in Tomiyama (1985) and Makris (2001). In this framework, the solution is found by implementing the following procedure. Splitting the objective function (7), present-value welfare equals the sum of the sub-streams of utilities obtained in the two phases, $V = V_1 + V_2$, where

$$V_1 = \int_0^T U(C_1(t))\, e^{-\rho t} dt, \tag{8}$$

$$V_2 = \int_T^\infty U(C_2(t))\, e^{-\rho t} dt. \tag{9}$$

The first step consists of optimizing phase 2 by finding the paths of consumption and knowledge $\{C_2(t), A(t)\}_T^\infty$ that maximize V_2 taking the switching time T as given. In the second step, we derive the set of conditions that are necessary for optimality during phase 1, for a given terminal time T. In the third step, we complete the set of optimality conditions by including the determination of the instant $T = T^*$ in which it is optimal to switch from the resource-based to the solar-based technology. This allows us to obtain the paths of consumption, knowledge and resource use $\{C_1(t), A(t), R(t)\}_0^{T^*}$ and $\{C_2(t), A(t)\}_{T^*}^\infty$ that maximize social welfare (7). For the sake of exposition, the discussion in the next section will be mainly technical. A more intuitive discussion about the economic consequences of backstop technology adoption will be provided in section 4.

3 The Optimal Control Problem

3.1 The solar-based economy

We begin by solving the social sub-problem of phase 2, i.e. after the backstop technology has been adopted. This problem consists of maximizing V_2 subject to the aggregate constraint (3), the solar-based technology (2), and the knowledge accumulation rule (4) in each $t \in (T, \infty)$, taking the available knowledge stock $A(T)$ as given, and holding T fixed. Obviously, this part of the solution is relevant only if the solar-based technology is actually adopted - that is, only if T is finite. Given this pre-condition, we have a standard infinite-horizon problem associated with the present-value Hamiltonian

$$H_2(t) = U(C_2(t)) e^{-\rho t} + \mu_2(t) \psi A(t) d_2(t), \tag{10}$$

where μ_2 is dynamic multiplier associated with the accumulation law (4). As shown in the Appendix, the necessary conditions for optimality are

$$\begin{aligned} \mu_2 \psi &= U'(C_2) \cdot e^{-\rho t} (Y_2/A), && (11) \\ -\dot{\mu}_2 &= (1 - d_2) \cdot U'(C_2) \cdot e^{-\rho t} (Y_2/A) + \mu_2 \psi d_2, && (12) \\ 0 &= \lim_{t \to \infty} A(t) \mu_2(t) e^{-\rho t}, && (13) \end{aligned}$$

and the following results hold:

Lemma 1 *(Solar-based economy, T given) In phase 2, the optimal propensity to consume equals*

$$c_2^* = 1 - (\psi - \rho) / (\psi \sigma) \tag{14}$$

in each $t \in (T, \infty)$. Output, consumption and knowledge grow at the constant rate

$$\dot{Y}_2(t) / Y_2(t) = \dot{C}_2(t) / C_2(t) = \dot{A}(t) / A(t) = \frac{1}{\sigma} (\psi - \rho) \tag{15}$$

from $t = T$ onwards. The optimal path is well-defined if and only if parameters satisfy

$$\psi (1 - \sigma) < \rho < \psi. \tag{16}$$

Lemma 1 shows that the solar-based economy exhibits a constant growth rate. It can be shown that the absence of transitional dynamics is due the linear accumulation function (4) previously assumed - see Rivera-Batiz and Romer (1991), Acemoglu (2002: sect.4). Expression (15) shows that consumption and output evolve according to the standard Keynes-Ramsey rule, where ψ (the rate of return from knowledge-improving investment) is the implicit interest rate of the economy. Restriction (16) is necessary and sufficient to have $c_2^* \in (0, 1)$, and also guarantees a strictly positive growth rate ($\psi > \rho$). Notice that the absence of transitional dynamics allows us to obtain closed-form solutions for all the endogenous variables during the second phase. In particular, consumption levels are given by[3]

$$C_2(t) = c_2^* \alpha A(T) (mG)^\gamma N^{1-\gamma} e^{(1/\sigma)(\psi - \rho)(t - T)}, \tag{17}$$

Equation (17) shows that consumption levels in the solar-based economy depend on the knowledge stock available at the beginning of the second phase. The optimal level of $A(T)$ is determined by the optimality conditions that characterize the behavior of the resource-based economy, as shown below.

3.2 The resource-based economy

The optimization problem in phase 1 consists of maximizing (8) subject to the aggregate constraint (3), the accumulation rule (4), and the natural resource constraint (5). Since resource extraction must be optimized, the path of the rate of resource use $R(t)$ represents an additional control variable for the social planner. The initial stocks, $A(0) = A_0$ and $S(0) = S_0$, are exogenously given, and the present-value Hamiltonian associated with this problem is

$$H_1(t) = U(C_1(t))\, e^{-\rho t} + \mu_1(t)\, \psi A(t)\, d_1(t) - \lambda(t)\, R(t), \tag{18}$$

where μ_1 and λ are the dynamic multipliers associated with the accumulation law (4) and the resource constraint (5), respectively. It should be noticed that, in the present problem, the terminal state to be imposed on the knowledge stock differs from the usual transversality condition $\mu_1(T)\, A(T) = 0$. The reason is that knowledge can be transferred to the solar-based economy, being further exploited during phase 2. If the solar-based technology is adopted in finite time, there is an implicit 'bequest' between the two phases, and the amount of knowledge left by the resource-based economy at the terminal date T is optimally chosen only if the effects of $A(T)$ on *second-phase* welfare are taken into account. In other words, the optimal path of knowledge accumulation incorporates the fact that, after time T, the same knowledge stock will be used for a different purpose - i.e. to complement solar-based energy in production. This reasoning has precise formalizations in optimal control theory (Tomyiama, 1985; Makris, 2001):

Lemma 2 *In the sub-problem of phase 1, the terminal conditions*

$$\lim_{t\to\infty} \mu_1(t)\, A(t) = 0 \qquad \text{if } T = \infty, \tag{19}$$

$$\lim_{t\to T^-} \mu_1(t) = \lim_{t\to T^+} \mu_2(t) \qquad \text{if } T < \infty, \tag{20}$$

are necessary for optimality.

Lemma 2 can be interpreted as follows. When there is no technology switching, $T = \infty$, the sub-problem collapses to a standard infinite horizon problem where the optimal path of knowledge accumulation is characterized by the transversality condition (19). When the solar-based technology is adopted, the amount of knowledge that the resource-based economy leaves for future use is optimally chosen only when (20) is satisfied. The intuition is that the dynamic multipliers μ_1 and μ_2 represent, in each phase, the marginal social value of an extra-unit of knowledge: condition (20) states that the optimal level of knowledge at the switching instant, $A(T)$, must be such that the marginal cost of accumulation in phase 1 equals the marginal benefit from knowledge exploitation in phase 2.

Before applying Lemma 2, it is possible to characterize the solution to the first-phase optimization problem as follows. Assuming that the optimal path of the propensity to consume is interior, $c_1(t) \in (0,1)$ in each $t \in [0,T)$, the optimal path is characterized by the following

Lemma 3 *(Phase 1, optimality conditions for given T) In the resource-based economy, the conditions*

$$\begin{aligned} \mu_1(t)\,\psi A(t) &= U'(C_1(t)) \cdot e^{-\rho t} Y_1(t), && (21)\\ \lambda(t)\,R(t) &= U'(C_1(t)) \cdot (1 - d_1(t))\,\delta Y_1(t)\,e^{-\rho t}, && (22)\\ \dot{\mu}_1(t) &= -\psi \mu_1(t), && (23)\\ \dot{\lambda}(t) &= 0, && (24)\\ S_0 &= \int_0^T R(t)\,dt, && (25) \end{aligned}$$

are necessary for optimality, where (21)-(24) are valid in each $t \in [0,T)$.

Equations (21)-(24) result from the usual optimality conditions, and do not require comment. Equation (25) follows from the transversality condition on the resource stock, and establishes that the initial stock must equal the sum of resource-use flows extracted during the first phase. In other words, the whole resource stock must be exhausted by the end of phase 1, since leaving unexploited resources in the ground would be sub-optimal.

The general implication of Lemma 3 is that consumption and resource use exhibit constant growth rates during phase 1. As shown in the Appendix, the resource-based economy displays

$$\begin{aligned} \frac{\dot{C}_1(t)}{C_1(t)} &= \frac{\psi - \rho(1+\delta)}{\sigma(1+\delta) - \delta}, && (26)\\ \frac{\dot{R}(t)}{R(t)} &= -\frac{\rho - (1-\sigma)\psi}{\sigma(1+\delta) - \delta} = -\phi, && (27) \end{aligned}$$

in each $t \in [0,T)$, where we have defined the constant

$$\phi \equiv \frac{\rho - (1-\sigma)\psi}{\sigma(1+\delta) - \delta} > 0 \qquad (28)$$

in order to represent the speed of resource depletion $-\dot{R}/R$ in a more compact way.[4] Results (26)-(27) are independent of the choice of the optimal switching time. With respect to consumption dynamics, it may be noticed that expression (26) differs from the Keynes-Ramsey rule (15) holding in phase 2: the two expressions coincide only if $\delta = 0$. The intuition is that, during phase 1, the economy is constrained by the non-renewable resource stock. As the exhaustible resource is exploited, increased scarcity is compensated by accumulating knowledge. This implies that resource productivity matters for consumption-saving decisions, and the elasticity parameter δ affects the growth rate of consumption during phase 1.

With respect to resource use dynamics, it is possible to derive a closed-form solution for the optimal extraction plan: integrating (27) over the interval $t \in [0, T)$, and substituting (25), we have

$$R(0) = \frac{S_0 \phi}{1 - e^{-\phi T}} \text{ and } R(t) = R(0) e^{-\phi t}. \tag{29}$$

The first expression in (29) shows that the initial rate of resource use $R(0)$ increases with the size of the initial stock S_0, and decreases with the length of the first phase T.

While (26)-(27) provide the basis for analyzing the dynamics of the resource-based economy, the optimal paths of consumption and knowledge during phase 1 are not determined until we impose the terminal conditions stated in Lemma 2. In this regard, we have to distinguish between the limiting case $T = \infty$, and the finite switching-time case $0 < T < \infty$.

Case $T = \infty$. If the solar-based technology is never adopted, we have $T = \infty$, and the accumulation of knowledge is subject to the transversality condition (19). In this case, the economy is permanently resource-based, and is characterized by the following dynamics:

Lemma 4 *(Phase 1, optimal path without switching) If $T = \infty$, the optimal propensity to consume equals*

$$c_1^*(t) = \frac{1}{\psi} \cdot \frac{\rho - \psi(1-\sigma)}{\sigma(1+\delta) - \delta} \tag{30}$$

in each $t \in [0, \infty)$. From (27), we have $\dot{R}(t)/R(t) = -\psi c_1^* < 0$. Output, consumption and knowledge grow at the constant rates

$$\dot{Y}_1(t)/Y_1(t) = \dot{C}_1(t)/C_1(t) = \frac{\psi - \rho(1+\delta)}{\sigma(1+\delta) - \delta}, \tag{31}$$

$$\dot{A}(t)/A(t) = \psi(1 - c_1^*), \tag{32}$$

in each $t \in [0, \infty)$. This path is well-defined if and only if parameters satisfy

$$\sigma(1+\delta) > \delta \text{ and} \tag{33}$$

$$\psi(1-\sigma) < \rho < \psi[1 - \delta(1-\sigma)] \tag{34}$$

Lemma 4 shows that, if the solar-based technology is never implemented, the resource-based economy exhibits a constant growth rate in each instant. In particular, the consumption propensity is constant over time, and is affected by the degree of resource dependence.

Case $0 < T < \infty$. If the solar-based technology is adopted in finite time, instead, we have $0 < T < \infty$, and the optimal path of knowledge is subject to the terminal condition (20). In this case, the economy must satisfy (see Appendix)

$$U'(C_1(T)) Y_1(T) = U'(C_2(T)) Y_2(T) \tag{35}$$

and the optimal paths of output and knowledge in phase 1 have to be determined simultaneously with the optimal switching time $T = T^*$, given that T^* is finite. It will be shown later that the characteristics of phase 1 with a finite switching time T are identical to those stated in Lemma 4, provided that T is optimally chosen:[5] imposing (35) together with the condition for optimal switching time, we re-obtain (30)-(34) over the finite interval $t \in [0, T)$ - see Lemma 6 below. Given this claim of observational equivalence between the cases $T = \infty$ and $0 < T < \infty$, it is possible to make two general remarks regarding phase 1.

The first remark is that, in the resource-based economy, sustained development is not a priori guaranteed. From (31), consumption and output grow at a positive rate if and only if

$$\frac{\psi}{1+\delta} > \rho. \tag{36}$$

For a given discount rate ρ, sustained growth in the first phase requires a moderate degree of resource dependence (low δ) and a sufficiently high productivity of investment (high ψ). If (36) is violated, the negative effect of resource depletion ($\dot{R} < 0$) is stronger than the positive effect of knowledge accumulation ($\dot{A} > 0$), and this implies declining time paths for output and consumption. Inequality (36) may indeed be considered an endogenous-growth variant of the sustainability condition derived in Stiglitz (1974).[6]

The second remark is that the economy exhibits different growth rates in the two phases. More precisely, *the solar-based economy grows faster than the resource-based economy*: from (15) and (31), the growth differential equals[7]

$$\frac{\dot{Y}_2}{Y_2} - \frac{\dot{Y}_1}{Y_1} = \frac{\delta}{\sigma} \cdot \frac{\rho - \psi\,(1-\sigma)}{\sigma\,(1+\delta) - \delta} > 0, \tag{37}$$

As may be construed, result (37) is determined by the constraint represented by resource scarcity. While the solar-based economy fully exploits the accumulation of knowledge, the resource-based economy exhibits a lower growth rate because the rate of resource extraction $R\,(t)$ declines over time. If resources were not essential in the first phase ($\delta = 0$), the two economies would grow at the same, balanced rate $\sigma^{-1}\,(\psi - \rho)$ determined by knowledge accumulation.

3.3 Optimal switching time

The third step of the solution to the social problem is the determination of the optimal timing of backstop technology adoption, T^*. In what follows, we will use a standard terminology. The optimal-timing problem exhibits an *interior solution* if $0 < T^* < \infty$, i.e. the solar-based technology is adopted in finite time but not immediately ($T^* > 0$). The alternatives are represented by the corner solutions $T^* = 0$ and $T^* = \infty$. In the first case, the optimal policy is that of *immediate adoption*, whereas $T^* = \infty$ represents no adoption - or equivalently, a *permanent delay* in the implementation of the solar-based technology.

The nature of the solution T^* can be clarified as follows. Given the two sub-problems of phase 1 and phase 2, denote by $\tilde{C}_1\,(t;T)$ and $\tilde{C}_2\,(t;T)$ the time paths of consumption that would be optimal in the two phases for a given switching time T. Using T as an unknown parameter, definitions (8)-(9) imply that the welfare levels associated with the two phases can be expressed as indirect welfare functions that depend on switching time:

$$V_1(T) = \int_0^T U\left(\tilde{C}_1(t;T)\right) e^{-\rho t} dt \text{ and } V_2(T) = \int_T^\infty U\left(\tilde{C}_2(t;T)\right) e^{-\rho t} dt. \tag{38}$$

From (38), total present-value welfare can be written as $V(T) = V_1(T) + V_2(T)$. Hence, the optimal timing of technology switching T^* is the instant in which the adoption of the solar-based yields the maximum present-value welfare over the entire time-horizon,

$$T^* = \arg \max_{T \in [0,\infty)} \{V(T) = V_1(T) + V_2(T)\}. \tag{39}$$

Under fairly general conditions, $V(T)$ is defined and finite in T, and differentiable at the switching instant (Seierstad and Sydsaeter, 1987). If the indirect function $V(T)$ is well-behaved - i.e. hump-shaped at least locally - problem (39) exhibits an interior maximum $0 < T^* < \infty$, and the solution is characterized by the first order condition $dV(T)/dT = 0$, i.e.

$$\frac{dV_1(T)}{dT} = -\frac{dV_2(T)}{dT}. \tag{40}$$

On the one hand, condition (40) represents an intuitive criterion: given a two-phase control problem, the optimal switching time is the instant in which the marginal welfare benefit from increasing the length of one phase equals the marginal welfare cost of reducing the length of the other phase. On the other hand, condition (40) is necessary and sufficient for an optimum only if $V(T)$ is well behaved: since the shape of $V(T)$ is not known a priori, the corner solutions of immediate adoption and permanent delay must be ruled out by showing that (40) is actually associated with a global maximum. In this regard, we will implement the following strategy. First, we show that there always exists a unique finite switching instant $T = T' > 0$ that satisfies condition (40). Second, we show that $V(T)$ is strictly concave, implying that $T^* = T'$ is indeed the solution to problem (39).

The behavior of $V(T)$ can be analyzed by applying optimal control theory. A well-known result establishes that, given a control problem with finite initial and terminal dates, the present-value Hamiltonian evaluated in the optimum equals (minus) the derivative of the value function with respect to the (initial) terminal date - see e.g. Seierstad and Sydsaeter (1987: Theorem 3.9). In the present context, this result is exploited as follows. Denote by $\bar{H}_1(T)$ the Hamiltonian function (18) evaluated at the switching time T along a path satisfying the optimality conditions (19)-(25). Symmetrically, denote by $\bar{H}_2(T)$ the Hamiltonian function (10) evaluated at the switching time T along a path satisfying the optimality conditions (11)-(13). Then, the derivatives of the indirect functions $V_1(T)$ and $V_2(T)$ are given by $dV_1(T)/dT = \bar{H}_1(T)$ and $dV_2(T)/dT = -\bar{H}_2(T)$, respectively. As a consequence, the derivative $dV(T)/dT = (dV_1(T)/dT) + (dV_2(T)/dT)$ equals the difference between the two Hamiltonians evaluated at the switching time. We can thus define the gap function

$$\Omega(T) \equiv \bar{H}_1(T) - \bar{H}_2(T) = dV(T)/dT, \tag{41}$$

and characterize the interior solutions to problem (39) by imposing the condition $\Omega(T) = 0$. The validity of this approach is confirmed by the results of Tomyiama (1985) and Makris

(2001), who show that, given a two-stage control problem, an interior solution for the optimal switching time $0 < T^* < \infty$ must satisfy the condition $\bar{H}_1(T^*) = \bar{H}_2(T^*)$.[8] Implementing this procedure yields the following

Lemma 5 *There exists a unique finite switching instant $T = T' > 0$ associated with $\Omega(T') = 0$. The function $\Omega(T)$ is strictly concave, and the solution to problem (39) is $T^* = T'$. The optimal switching instant is given by*

$$T^* = \frac{1}{\phi} \ln \left\{ 1 + \left[1 - (\delta/\sigma)(1-\sigma)\right]^{\frac{\sigma}{\delta(1-\sigma)}} \beta^{1/\delta} \right\} > 0, \tag{42}$$

where $\beta \equiv (nS_0\phi)^{\delta} N^{\gamma-\delta}\alpha^{-1}(mG)^{-\gamma}$. If preferences are logarithmic, $\sigma = 1$, the same results hold with an optimal switching time $T^* = (1/\phi)\ln\left[1 + (\beta/e)^{1/\delta}\right]$.

Lemma 5 is a crucial result of this paper. It shows that, given the assumptions made so far, the optimal timing of backstop technology adoption is unique, and can be expressed as a function of the model parameters. A numerical example that confirms Lemma 5 is described in Figure 1, where the indirect welfare function $V(T)$ and the gap function $\Omega(T)$ are obtained for a given set of parameter values[9]. The indirect welfare function achieves a maximum in $T^* = 16.3$, associated with the horizontal intercept of the gap function, $\Omega(T^*) = 0$.

We now have all the elements to characterize the optimal path of the economy in both phases. The following section describes the main results of the analysis, and discusses the economic consequences of backstop technology adoption.

4 Consequences of Technology Switching

On the basis of our previous results, the optimal path of the economy over the whole time horizon $t \in [0, \infty)$ can be described as follows. In the second phase, delimited by the interval $t \in (T^*, \infty)$, the optimal path is characterized by the conditions already stated in Lemma 1. In the first phase, delimited by the interval $t \in [0, T^*)$, the economy exploits exhaustible resources according to technology (1). The following Lemma establishes that, during phase 1, the economy exhibits balanced growth in each instant, and displays the same properties, in terms of growth rates and consumption/saving propensities, of the infinite-horizon resource-based economy described in Lemma 4:

Lemma 6 *Given an optimal switching time $T = T^*$, the resource-based economy follows an optimal path in which the optimal propensity to consume equals (30), output and consumption grow at the constant rate (31), and knowledge grows at the constant rate (32) in each $t \in [0, T^*)$. The optimal path is well-defined if and only if parameters satisfy (33)-(34).*

Lemmas 6 and equation (37) imply that, along the optimal path, the resource-based economy grows at slower rate with respect to the solar-based economy. As noted before, the reason is that the growth process in phase 1 is constrained by resource scarcity. The most interesting aspect is related to the immediate consequences of adopting the backstop technology:

Proposition 7 *The adoption of the solar-based technology implies discrete jumps in consumption, output and growth. The transition to the solar-based economy is characterized by sudden reductions in consumption and output levels,*

$$\begin{aligned} C_1 (T^*) / C_2 (T^*) &= \left\{ \sigma \left[\sigma - \delta (1-\sigma) \right]^{-1} \right\}^{\frac{1}{1-\sigma}} > 1, \\ Y_1 (T^*) / Y_2 (T^*) &= \left\{ \sigma \left[\sigma - \delta (1-\sigma) \right]^{-1} \right\}^{\frac{\sigma}{1-\sigma}} > 1, \end{aligned}$$

and, from (37), an immediate increase in the growth rate. If preferences are not logarithmic, $\sigma \neq 1$, technology adoption also implies a discrete jump in consumption/saving propensities,

$$c_1 (T^*) / c_2 (T^*) = \sigma \left[\sigma - \delta (1-\sigma) \right]^{-1}.$$

Proposition 7 is the main result of this paper. At the technical level, the presence of discrete jumps may seem surprising. In optimal control problems, an interior optimal path is characterized by the continuity of the Hamiltonian function over the whole time horizon, $t \in [0, \infty)$. In standard growth models, this is often taken to imply smooth dynamics in the control variables - e.g. consumption. Proposition 7 clarifies that similar continuity arguments are not robust, at least with respect to the present model: the 'general' Hamiltonian function of the social problem, $H(t) = H_1(t) + H_2(t)$, is indeed continuous in $t = T^*$, but this does not imply the smoothness of the optimal paths of consumption and output.[10] Apart from technical issues, however, the scope of Proposition 7 is determined by the underlying economic intuition, which can be summarized as follows.

The optimal switching time is characterized by a precise trade-off between level effects and growth effects. On the one hand, the solar-based economy grows faster than the resource-based economy. On the other hand, the adoption of the backstop technology induces a sudden reduction in consumption and output. The implicit reasoning is that, given the possibility of extracting exhaustible resources in the present, the traditional technology can be exploited in order to obtain higher output levels in the short run. Although the economy would grow faster under solar-based technologies, it is optimal to exploit the available resource for a while before switching to solar energy: higher consumption levels in the short run guarantee higher welfare with respect to the alternative policy of immediate adoption of the solar-based technology ($T = 0$). When the resource stock is optimally exhausted, the adoption of the backstop technology yields lower consumption, but this level effect is compensated (in terms of present-value welfare) by the higher growth rate that the economy enjoys from $t = T^*$ onwards.

Proposition 7 also suggests a remark on the role of preferences. While the direction of level and growth effects is unambiguous, the way in which consumption and saving *propensities* adjust to the new technology depends on the elasticity of intertemporal substitution. If $\sigma < 1$ we have $c_1^* > c_2^*$, i.e. the propensity to consume is suddenly reduced by the adoption of the backstop technology. The opposite phenomenon arises when $\sigma > 1$, which implies an upward jump in the consumption propensity, $c_1^* < c_2^*$. When preferences are logarithmic, $\sigma = 1$, the adoption of the backstop technology has no effects on consumption and saving propensities. Nonetheless, there are discrete jumps in consumption, output and growth rates: when $\sigma = 1$, the size of the reduction in consumption levels is $C_1(T^*)/C_2(T^*) = e^{\delta}$, so that the magnitude of the level effects of technology adoption increases exponentially with the degree of resource dependence.[11]

Another remark is related to the behavior of marginal productivities along the optimal path. In the present model, the (social) profitability rates associated with the primary energy sources are represented by $\partial Y_1/\partial R$ and $\partial Y_2/\partial G$. As shown in the Appendix, the optimal switching time is characterized by

$$\frac{\partial Y_1(T^*)/\partial R(T^*)}{\partial Y_2(T^*)/\partial G} = \left(\frac{n^{\delta} N^{\gamma-\delta}}{\alpha m^{\gamma}}\right)^{1/\delta} \frac{\delta G^{1-\frac{\gamma}{\delta}}}{\gamma} \left\{\frac{\sigma - \delta(1-\sigma)}{\sigma}\right\}^{\frac{\sigma(1-\delta)}{\delta(1-\sigma)}}. \tag{43}$$

Expression (43) shows that the marginal productivities of primary energy sources are generally different at time T^*. Even assuming a convenient set of parameters - e.g. identical production shares $\gamma = \delta$ and unit productivity indices $\alpha = m = n = 1$ - the right hand side of (43) differs from unity. This is due to the term in curly brackets, which indeed determines the size of consumption and output jumps (cf. Proposition 7). As emphasized in the next section, the fact that marginal productivities of resource-based and solar-based energy do not generally coincide in the switching instant is an important difference with respect to the early literature on backstop technology adoption.

All the above conclusions can be verified by numerical simulation. Two examples are reported in Table 1, and graphically described in Figure 2. Except for σ, the parameter values are the same used in Figure 1 - see footnote 9.

	$\sigma = 0.8$	$\sigma = 1.2$
Output: $Y_1(T^*)$, $Y_2(T^*)$	37.4 , 28.9	34.5 , 27.0
Cons. Propensity: c_1^* , c_2^*	0.62 , 0.58	0.69 , 0.72
Consumption: $C_1(T^*)$, $C_2(T^*)$	23.2 , 16.8	23.9 , 19.5
Growth Rates: $\dot{Y}_1/Y_1$, $\dot{Y}_2/Y_2$	1.3% , 2.5%	0.8% , 1.6%
Switching Time	$T^* = 16.2$	$T^* = 16.4$

Table 1. Optimal jumps: simulation results (see also Figure 2).

With $\sigma = 0.8 < 1$, we obtain an optimal switching time $T^* = 16.2$, and a downward jump in the consumption propensity associated with the adoption of the solar-based technology. With $\sigma = 1.2 < 1$, technology adoption is slightly delayed, and implies a sudden increase in the propensity to consume. In both scenarios, the values of the marginal productivities of primary energy sources are substantially different. For example, with $\sigma = 0.8$ we obtain $\partial Y_1/\partial R = 0.21$ and $\partial Y_2/\partial G = 8.66$ at time $t = T^*$.

5 Connections with previous literature

As explained in the Introduction, the main references for the present analysis are represented by the early studies of backstop technology adoption - pioneered by Hoel (1978), Dasgupta and Stiglitz (1981), and Dasgupta et al. (1982) - and the more recent endogenous growth models where natural resources are essential inputs in production (Barbier, 1999; Sholz and Ziemes, 1999; Aznar-Marquez and Ruiz-Tamarit, 2005).

The early literature tackles the issue of technology switching by implementing a price-based criterion: the backstop technology is adopted when the resource-intensive method becomes as expensive as solar-based techniques (Hoel, 1978). Variants of the basic model include different market structures (Dasgupta et al. 1982), uncertainty (Dasgupta and Stiglitz, 1981; Hung and Quyen, 1993), and pollution (Tahvonen, 1997), and other features that modify the timing of technology switching, but the underlying criterion remains price-based. With respect to these contributions, our analysis shows that results are substantially modified when (i) the optimal timing of backstop technology adoption obeys a welfare-based criterion, and (ii) the profitability of competing technologies is determined by the development paths generated by endogenous growth mechanisms. In our framework, the cost-benefit analysis pursued by the planner takes into account all the general-equilibrium effects that characterize the two phases of economic development: when production possibilities are constrained by resource scarcity (phase 1) the economy develops at slower rates, whereas solar-based technologies guarantee sustained and faster growth (phase 2). Under these circumstances, a forward-looking planner will exploit the resource-based technology for obtaining high levels of output in the short run, and then switch to solar-based techniques to generate faster growth in the medium-long run. The optimal switching time is not characterized by the equality between the marginal productivities of the two energy sources.

With respect to the endogenous-growth literature, the vast majority of sustainability models postulate a continuous process of resource-augmenting technical progress whereby resource inputs are progressively substituted by knowledge-type capital (Barbier, 1999) or expanding varieties of intermediate products (Sholz and Ziemes, 1999). This process is smooth, and the underlying reasoning is that knowledge progressively substitutes exhaustible resources in production. This paper extended this class of models to include a backstop technology, and showed that the transitional dynamics are anything but smooth: the optimal path exhibits discrete jumps in endogenous variables. Due to the contrasting effects of technology adoption on levels and growth rates, the implementation of new technologies is optimal even though the associated current benefits in terms of consumption (evaluated at the optimal switching instant) are substantially lower than those generated by traditional technologies. This result appears consistent with concrete reality, since non-traditional energy sources like wind and solar power are regarded as less profitable nowadays, but are more likely to guarantee sustainable growth in the future.

Our analysis suggests a number of extensions that can be implemented in an endogenous growth setting. A natural question relates to the effects of market failures on the optimal timing of structural change. Endogenous growth models typically assume that non-decreasing returns hinge on the presence of externalities - e.g. Aznar-Marquez and Ruiz-Tamarit (2005). In this framework, decentralized competitive equilibria are characterized by intertemporal allocations that differ from the social optimum studied here. This suggests studying the role of externalities and optimal policies in order to optimize the timing of backstop technology adoption.

6 Conclusion

This paper studied the optimal timing of backstop technology adoption in a two-phase endogenous growth model. Aggregate production requires energy, initially produced by means of exhaustible resources. A backstop technology represented by solar-based energy is available, and a benevolent social planner decides whether and when to abandon traditional oil-based energy in favor of the new technology. It has been shown that the optimal path is characterized by a trade-off between the level effects produced by the resource-based technology and the growth-enhancing effects induced by the solar-based technology. The economy exploits the resource stock to obtain high consumption levels in the short run, and then switches to solar-based techniques in order to achieve faster growth in the medium-long run. There always exists an interior solution to the optimal timing problem, characterized by discrete jumps in consumption and output: the adoption of the backstop technology implies a sudden reduction in consumption and output levels, an increase in the growth rate, as well as instantaneous adjustments in consumption/saving propensities. The marginal productivities of the competing energy sources evaluated at the instant of technology switching are generally different.

The implementation of backstop technologies is optimal even though their current benefits in terms of consumption are substantially lower than those generated by traditional technologies. This result appears consistent with concrete reality, since non-traditional energy sources like wind and solar power are regarded as less profitable nowadays, but are more likely to guarantee sustainability in the future. More generally, the analysis shows that welfare-based criteria, in conjunction with endogenous-growth mechanisms, yield substantial differences with respect to the results of the early literature on backstop technologies: the optimal switching time is determined by a more complete forward-looking criterion that takes into account the future growth effects of backstop technology adoption. Extending this framework to analyze decentralized economies in which the timing of technology switching is affected by the presence of market failures is the main suggestion for future research.

Appendix

Derivation of (11)-(13). From the aggregate constraint of the economy and technology (2), consumption equals $C_2 = (1-d_2)\,Y_2 = (1-d_2)\,\alpha A\,(mG)^{\gamma}\,N^{1-\gamma}$. The second-phase problem can be thus specified as

$$\max_{\{d_2\}_{t=T}^{\infty}} V_2 = \int_T^{\infty} U\left[(1-d_2)\,\alpha A\,(mG)^{\gamma}\,N^{1-\gamma}\right] e^{-\rho t} dt$$

subject to $\dot{A} = \psi A d_2$, with $A(T)$ given, and the saving propensity d_2 acting as control variable. The present-value Hamiltonian (10) can be written as

$$H_2 = U\left[(1-d_2)\,\alpha A\,(mG)^{\gamma}\,N^{1-\gamma}\right] e^{-\rho t} + \mu_2 \psi A d_2. \tag{A1}$$

Equation (11) is given by $\partial H_2/\partial d_2 = 0$, (12) is the co-state equation $\partial H_2/\partial A = -\dot{\mu}_2$, and (13) is the standard transversality condition. ■

Proof of Lemma 1. Denoting by $\hat{x} \equiv \dot{x}/x$ the instantaneous growth rate of the generic variable $x(t)$, time-differentiation of (11) yields

$$\hat{\mu} = \hat{Y}_2 - \hat{A} + \hat{U}' - \rho = \hat{U}' - \rho, \tag{A2}$$

where the last term comes from the fact that $\hat{Y}_2 = \hat{A}$. Plugging (11) in (12), we obtain

$$\hat{\mu}_2 = -\psi. \tag{A3}$$

Combining (A3) with (A2) and $\hat{U}' = -\sigma \hat{C}_2$, we obtain

$$\hat{C}_2 = \sigma^{-1}(\psi - \rho). \tag{A4}$$

Substituting $\hat{C}_2 = \hat{c}_2 + \hat{Y}_2$ and $\hat{Y}_2 = \hat{A} = \psi d_2$ in (A4), and using $c_2 = 1 - d_2$, the optimal propensity to consume must satisfy

$$\hat{c}_2 = \psi c_2 + \sigma^{-1}(\psi - \rho) - \psi, \tag{A5}$$

Relation (A5) is globally unstable with a unique fixed point

$$\bar{c}_2 = 1 - \frac{\psi - \rho}{\psi \sigma}. \tag{A6}$$

By standard arguments, explosive dynamics of c_2 can be ruled out as they would lead to either negative consumption or negative output in finite time. The optimal propensity c_2^* is thus equal to $\bar{c}_2$ in each $t \in [T, \infty)$, which proves equation (14). A constant propensity to consume implies $\dot{Y}_2/Y_2 = \dot{C}_2/C_2 = \dot{A}/A$. Imposing $0 < c_2^* < 1$, we obtain restriction (16), which completes the proof. ■

Proof of Lemma 2. If the solar-based technology is never adopted, the first-phase problem reduces to a standard infinite-horizon optimal control. The transversality condition on the knowledge stock is (19), and does not require further comments. If the solar-based technology is adopted at some finite T, instead, the social problem belongs to the class of two-stage problems analyzed in Tomiyama (1985) and Makris (2001). Using the current notation, expression (20) corresponds to the optimality condition derived e.g. in Tomiyama (1985: Theorem 1, equation [15]). ■

Proof of Lemma 3. From (1), consumption in phase 1 equals $C_1 = (1 - d_1) Y_2 = (1 - d_1) A (nR)^{\delta} N^{1-\delta}$. The first-phase problem can be thus specified as

$$\max_{\{d_1, R\}_{t=0}^{T}} V_1 = \int_0^T U\left[(1 - d_1) A (nR)^{\delta} N^{1-\delta}\right] e^{-\rho t} dt$$

subject to $\dot{A} = \psi A d_1$, with $A(0) = A_0$ given, and to $\dot{S} = -R$, with $S(0) = S_0$ given. The saving propensity d_1 and the rate of resource use R act as control variables. The present-value Hamiltonian (18) can be written as

$$H_1 = U\left[(1 - d_1) A (nR)^{\delta} N^{1-\delta}\right] e^{-\rho t} + \mu_1 \psi A d_1 - \lambda R. \tag{A7}$$

Equations (21)-(22) are given by the first-order conditions $\partial H_1/\partial d_1 = 0$ and $\partial H_1/\partial R = 0$, respectively. The co-state condition $\partial H_1/\partial A = -\dot{\mu}_1$ yields

$$-\dot{\mu}_1 = (1 - d_1) \cdot U'(C_1) \cdot e^{-\rho t} (Y_1/A) + \mu_1 \psi d_1, \tag{A8}$$

where we can substitute (21) to obtain (23). Equation (24) is given by the co-state condition $\partial H_1/\partial S = -\dot{\lambda}$. The transversality condition on the resource stock reads

$$\lambda(T) S(T) = 0. \tag{A9}$$

Since $\lambda(t)$ is constant by (24), condition (A9) requires exhausting the whole resource stock at the end of the first phase, $S(T) = 0$. Integration of the dynamic law (5) between $t = 0$ and $t = T^*$ yields

$$S(T) = S_0 - \int_0^T R(t)\, dt.$$

Substituting $S(T) = 0$ in the above expression yields equation (25). ■

Derivation of (26)-(27). Since $\lambda(t)$ is constant by (24), time-differentiation of (22) yields

$$\hat{R} = \hat{U}' + \hat{c}_1 + \hat{Y}_1 - \rho = \hat{U}' + \hat{C}_1 - \rho, \tag{A10}$$

Time-differentiating (21), and eliminating $\hat{\mu}_1$ by means of (23), we have

$$\hat{U}' = \rho - \psi - \hat{Y}_1 + \hat{A}. \tag{A11}$$

Time-differentiating (1), we have $\hat{Y}_1 = \hat{A} + \delta \hat{R}$. Plugging this result in (A11), and substituting (A10), we obtain

$$\hat{U}'(1 + \delta) = \rho - \psi - \delta \hat{C}_1 + \delta \rho. \tag{A12}$$

Substituting $\hat{U}' = -\sigma \hat{C}_1$ in (A12) gives (26). Plugging (26) and $\hat{U}' = -\sigma \hat{C}_1$ in (A10), we obtain (27). Since (21)-(24) are valid in each $t \in [0, T)$ independently of whether T is finite or not, results (26)-(27) hold independently of whether the optimal switching time is finite or not. For future reference, notice that (26)-(27) imply a dynamic relation that must be satisfied by the optimal propensity to consume[12]:

$$\hat{c}_1(t) = \psi c_1(t) - \frac{\rho - \psi(1 - \sigma)}{\sigma(1 + \delta) - \delta}. \tag{A13}$$

Proof of Lemma 4. Suppose that $T = \infty$. The dynamic relation (A13) is globally unstable with a unique fixed point

$$\bar{c}_1 = \frac{1}{\psi} \cdot \frac{\rho - \psi(1 - \sigma)}{\sigma(1 + \delta) - \delta}. \tag{A14}$$

By standard arguments, explosive dynamics of $c_1(t)$ can be ruled out as they would lead to either negative consumption or negative output in finite time. The optimal propensity $c_1^*(t)$ is thus equal to $\bar{c}_1$ in each $t \in [T, \infty)$, which proves result (30). From (27) and (30), the depletion rate ϕ coincides with ψc_1^*, so that $\hat{R} = -\psi c_1^* < 0$. Given a constant propensity c_1^*, output grows at the same rate as consumption, and (26) implies (31). Equation (32) follows directly from (4). Restriction (33) and the first inequality in (34) guarantees $c_1^* > 0$ in (30), whereas $c_1^* < 1$ requires the respect of the second inequality in (34). ■

Derivation of (35). With slight abuse of notation, condition (20) can be re-written as

$$\mu_1(T) = \mu_2(T). \tag{A15}$$

Substituting (21) and (23) in (A15), we obtain (35).

Proof of Lemma 5. The first step of the proof is to derive an expression for the gap function (41). From the optimality conditions (21) and (11), we have $\mu_i \psi A d_i = U'(C_i) \cdot e^{-\rho t} Y_i d_i$ in each phase $i = 1, 2$. Substituting these conditions in (18) and (10), the Hamiltonians of the two sub-problems evaluated at the switching instant T read

$$\begin{aligned} H_1(T) &= e^{-\rho T}\left[U(C_1(T)) + U'(C_1(T))\, Y_1(T)\, d_1(T)\right] - \lambda(T)\, R(T), \\ H_2(T) &= e^{-\rho T}\left[U(C_2(T)) + U'(C_2(T))\, Y_2(T)\, d_2(T)\right], \end{aligned}$$

Substituting $\lambda(T)\, R(T) = \delta U'(C_1(T))\, C_1(T)\, e^{-\rho T}$ from (22), and using $c_i = 1 - d_i$ to substitute $U'(C_i)\, Y_i d_i = -U'(C_i)\, C_i + U'(C_i)\, Y_i$ in each phase $i = 1, 2$, the above expressions yield

$$H_1(T) = e^{-\rho T}\left[U(C_1) - U'(C_1)\, C_1 + U'(C_1)\, Y_1 - \delta U'(C_1)\, C_1\right], \tag{A16}$$
$$H_2(T) = e^{-\rho T}\left[U(C_2) - U'(C_2)\, C_2 + U'(C_2)\, Y_2\right], \tag{A17}$$

where all variables are evaluated at T. As shown in (35), the terminal condition for optimal knowledge accumulation (20) requires satisfying $U'(C_1(T))\, Y_1(T) = U'(C_2(T))\, Y_2(T)$ at the switching instant T. Taking the difference between (A16) and (A17), and substituting (35) in the resulting expression we obtain

$$H_1(T) - H_2(T) = e^{-\rho T}\left[U(C_1) - U'(C_1)\, C_1 (1 + \delta) - U(C_2) + U'(C_2)\, C_2\right], \tag{A18}$$

where consumption levels are evaluated at T. Notice that expression (A18) is also valid in the limiting case $\sigma = 1$, in which preferences become logarithmic. Now assume that σ differs from unity. Plugging $U(C_i) = \left(C_i^{1-\sigma} - 1\right)(1 - \sigma)^{-1}$ and $U'(C_i)\, C_i = C_i^{1-\sigma}$ in (A18), we can re-write the gap function $\Omega(T) \equiv \bar{H}_1(T) - \bar{H}_2(T)$ defined in (41) as

$$\Omega(T) = e^{-\rho T} \frac{\sigma}{1-\sigma}\left[C_1(T)^{1-\sigma} - C_2(T)^{1-\sigma} - (\delta/\sigma)(1 - \sigma)\, C_1(T)^{1-\sigma}\right]. \tag{A19}$$

Expression (A19) can be written in more convenient terms as follows. Rewrite the terminal condition (35) as

$$\frac{C_1(T)}{C_2(T)} = \left(\frac{Y_1(T)}{Y_2(T)}\right)^{\frac{1}{\sigma}} = \left[\frac{(nR(T))^{\delta}}{\alpha (mG)^{\gamma}} N^{\gamma-\delta}\right]^{\frac{1}{\sigma}}, \tag{A20}$$

where the last term is obtained by substituting technologies (1)-(2). From (29) we also know that $R(T) = S_0 \phi \left(e^{\phi T} - 1\right)^{-1}$. Substituting this equation in (A20), and collecting the constant terms in $\beta \equiv (n S_0 \phi)^{\delta} N^{\gamma-\delta} \alpha^{-1} (mG)^{-\gamma}$, we obtain

$$C_1(T) / C_2(T) = \beta^{1/\sigma} \left(e^{\phi T} - 1\right)^{-\delta/\sigma}. \tag{A21}$$

Plugging (A21) in (A19) to eliminate $C_1(T)$, the gap function reads

$$\Omega(T) = \frac{\sigma C_2(T)^{1-\sigma} e^{-\rho T}}{1-\sigma} \left\{ [1 - (\delta/\sigma)(1-\sigma)] \beta^{\left(\frac{1-\sigma}{\sigma}\right)} \left(e^{\phi T} - 1\right)^{-\frac{\delta(1-\sigma)}{\sigma}} - 1 \right\}. \tag{A22}$$

Clearly, $\Omega(T) = 0$ requires that the term in curly brackets equals zero. Imposing this condition, we have

$$e^{\phi T} = 1 + [1 - (\delta/\sigma)(1-\sigma)]^{\frac{\sigma}{\delta(1-\sigma)}} \beta^{1/\delta}. \tag{A23}$$

The left hand side of (A23) is a strictly increasing function of T, whereas the right hand side is a positive constant independent of T. As a consequence, there exists a unique value $T = T'$ satisfying the above equation. Taking the logarithms of both sides, and solving for the switching time, we obtain

$$T' \equiv \frac{1}{\phi} \ln \left\{ 1 + [1 - (\delta/\sigma)(1-\sigma)]^{\frac{\sigma}{\delta(1-\sigma)}} \beta^{1/\delta} \right\}, \tag{A24}$$

where T' is finite and strictly positive because restrictions (33)-(34) imply $\phi > 0$ and $[1 - (\delta/\sigma)(1-\sigma)] > 0$.[13] Expression (A24) thus gives the unique switching instant $T = T'$ associated with the critical condition $\Omega(T') = \bar{H}_1(T') - \bar{H}_2(T') = 0$. We now prove that $T = T'$ is the maximum of $V(T)$ by showing that $V(T)$ is strictly increasing in any $T < T'$ and strictly decreasing in any $T > T'$. Rewrite (A22) as

$$\Omega(T) = \sigma C_2(T)^{1-\sigma} e^{-\rho T} \cdot \left[\frac{f(T) - 1}{1-\sigma} \right], \tag{A25}$$

where $f(T) \equiv [1 - (\delta/\sigma)(1-\sigma)] \beta^{\left(\frac{1-\sigma}{\sigma}\right)} \left(e^{\phi T} - 1\right)^{-\frac{\delta(1-\sigma)}{\sigma}}$. The sign of $\Omega(T)$ is determined by the term in square brackets in (A25). First suppose that $\sigma < 1$. In this case, $f(T)$ is strictly decreasing in T. Since $f(T') = 1$ by (A23), we have $f(T'') > 1$ for any $T'' < T'$, and $f(T''') < 1$ for any $T''' > T'$. Given $\sigma < 1$, this implies $\Omega(T'') > 0$ for any $T'' < T'$, and $\Omega(T''') < 0$ for any $T''' > T'$. Now suppose that $\sigma > 1$ instead. In this case, $f(T)$ is strictly increasing in T, so that $f(T'') < 1$ for any $T'' < T'$, and $f(T''') > 1$ for any $T''' > T'$. Given $\sigma > 1$, this implies again $\Omega(T'') > 0$ for any $T'' < T'$, and $\Omega(T''') < 0$ for any $T''' > T'$. These results imply that $V(T)$ is strictly concave, and that $T = T'$ is the maximum of $V(T)$. We can thus set $T^* = T'$ in (A25) to obtain (42). The proof of Lemma 5 is completed by considering logarithmic preferences, $\sigma = 1$. Going back to equation (A18), we can substitute $U(C_i) = \ln C_i$ and $U'(C_i) = C_i^{-1}$ in both phases $i = 1, 2$ to obtain

$$H_1(T) - H_2(T) = e^{-\rho T} \left[\ln \left(\frac{C_1(T)}{C_2(T)} \right) - \delta \right], \quad \sigma = 1. \tag{A26}$$

The terminal condition (A21) reduces to

$$C_1(T)/C_2(T) = \beta \left(e^{\phi T} - 1\right)^{-\delta}, \quad \sigma = 1. \tag{A27}$$

Plugging (A27) in (A26), and imposing $H_1(T^*) - H_2(T^*) = 0$, we obtain $e^{\phi T^*} = 1 + (\beta/e)^{1/\delta}$, from which $T^* = (1/\phi) \ln \left[1 + (\beta/e)^{1/\delta} \right]$. ■

Proof of Lemma 6. From the proof of Lemma 5, the optimal switching time T^* is characterized by $\Omega(T^*) = 0$. From (A19), setting $\Omega(T^*) = 0$ implies

$$C_1(T^*)/C_2(T^*) = \left\{\sigma[\sigma - \delta(1-\sigma)]^{-1}\right\}^{\frac{1}{1-\sigma}}. \tag{A28}$$

Combining (A28) with the condition for optimal knowledge accumulation (A20), we have

$$Y_1(T^*)/Y_2(T^*) = \left\{\sigma[\sigma - \delta(1-\sigma)]^{-1}\right\}^{\frac{\sigma}{1-\sigma}}. \tag{A29}$$

Taking the ratio between (A28) and (A29), the optimal ratio between consumption propensities is

$$c_1(T^*)/c_2(T^*) = \sigma[\sigma - \delta(1-\sigma)]^{-1}. \tag{A30}$$

From Lemma 1, we can use (14) to substitute $c_2(T^*) = c_2^*$ in (A30), obtaining

$$c_1(T^*) = \frac{1}{\psi} \cdot \frac{\rho - \psi(1-\sigma)}{\sigma(1+\delta) - \delta}. \tag{A31}$$

Recalling the derivation of equations (26)-(27), the optimal path of the consumption propensity in the resource-based economy must satisfy the dynamic relation (A13). Expression (A31) implies that the optimal consumption propensity at the switching instant, $c_1(T^*)$, must be equal to the steady-state point $\bar{c}_1$ of (A13) - see equation (A14) above. Since (A13) is globally unstable, the only way to satisfy (A31) is to set $c_1(t) = \bar{c}_1$ in each instant $t \in [0, T^*)$. As a consequence, the optimal path is characterized by a constant propensity to consume c_1^* given by (30).[14] From (26), this implies that output and consumption grow at the constant rate (31), and knowledge grows at the constant rate (32) in each $t \in [0, T^*)$. Recalling that (27) and (A31) imply $\dot{R}/R = -\phi = -\psi c_1^* < 0$, the optimal path is well-defined if and only if parameters satisfy (33)-(34). ■

Proof of Proposition 7. The equations appearing in Proposition 7 are given by (A28), (A29) and (A30), respectively. ■

Derivation of (43). By definition, the ratio between the marginal productivities $\partial Y_1/\partial R$ and $\partial Y_2/\partial G$ equals $(\partial Y_1/\partial R)/(\partial Y_2/\partial G) = (\delta Y_1 G)/(\gamma Y_2 R)$. Plugging this expression in (A29) we have

$$\frac{\partial Y_1(T^*)/\partial R(T^*)}{\partial Y_2(T^*)/\partial G} = \frac{\delta G}{\gamma R(T^*)}\left\{\sigma[\sigma - \delta(1-\sigma)]^{-1}\right\}^{\frac{\sigma}{1-\sigma}}. \tag{A32}$$

From (29) and (A23), we can respectively substitute $R(T^*) = S_0\phi\left(e^{\phi T^*} - 1\right)^{-1}$ and $e^{\phi T^*} - 1 = [1 + (\delta/\sigma)(1-\sigma)]^{\frac{\sigma}{\delta(1-\sigma)}}\beta^{1/\delta}$, to obtain

$$\frac{\partial Y_1(T^*)/\partial R(T^*)}{\partial Y_2(T^*)/\partial G} = \beta^{1/\delta}\frac{\delta G}{\gamma S_0 \phi}\left\{\frac{\sigma - \delta(1-\sigma)}{\sigma}\right\}^{\frac{\sigma(1-\delta)}{\delta(1-\sigma)}}.$$

Substituting the definition of β from Lemma 5, we obtain expression (43).

Notes

[1]In the context of exhaustible resources, the sustainability condition derived by Stiglitz (1974) establishes that non-declining consumption in the long run requires the utility discount rate be not less than the rate of resource-saving technical progress. The same condition can be shown to be valid in endogenous-growth models where both the speed and the direction of technical progress are driven by human decisions - see Di Maria and Valente (2008). If the natural resource is renewable, the Stiglitz (1974) condition must be augmented by the marginal rate of resource regeneration - see Valente (2005) for a generalization of the neoclassical framework - and can be expressed, in an endogenous-growth setting, in terms of the rate of resource use, as shown by Aznar-Marquez and Ruiz-Tamarit (2005).

[2]A possible micro-foundation of (4) is that used in Barro and Sala-i-Martin (2004: Ch.6), where A is the number of varieties of intermediate goods, and $\dot{A} = \nu D$ in a model where output is linear in the number of varieties, i.e. $Y = \phi A$. In this case, defining $\psi = \nu\phi$ yields $\dot{A} = \psi dA$, and the economy exhibits balanced growth in each instant. The same reasoning can be applied to model with resource extraction (Valente, 2008: sec.4.3).

[3]By integration of the accumulation law (4), we have $A(t) = A(T)\, e^{\psi d_2^*(t-T)}$, where $d_2^* \equiv 1 - c_2^*$ implies $A(t) = A(T)\, e^{(1/\sigma)(\psi-\rho)(t-T)}$. Substituting this result in $C_2(t) = c_2^* Y_2(t) = c_2^* \alpha A(t)(mG)^{\gamma} N^{1-\gamma}$, we obtain equation (17) in the text.

[4]As shown below (cf. Lemmas 4 and 6), optimality requires a declining rate of resource over time, so that parameters must satisfy the restriction $\phi > 0$. This result is in line with conclusions of the early literature on capital-resource models - e.g. Dasgupta and Heal (1974).

[5] The only situation in which c_1^* does not have to be determined simultaneously with the optimal switching time $T = T^*$ is the case of logarithmic preferences, $\sigma = 1$. In fact, when $U(C_i) = \ln C_i$, the terminal condition (35) implies $c_1^*/c_2^* = 1$ independently of the switching time T. In this case, we have $c_2^* = \rho/\psi$ from (14), and therefore equal propensities $c_1^* = c_2^* = \rho/\psi$ in both phases independently of the switching time T.

[6]Assuming capital-resource technology of the type $Y = AK^{1-\delta}R^{\delta}$ with an exogenous rate of Hicks-neutral technical progress $\dot{A}/A = \nu$, Stiglitz (1974) showed that output and consumption are asymptotically increasing if $\nu/\delta > \rho$, where δ is the resource share in production.

[7]Both the numerator and the denominator in (37) are strictly positive by (33)-(34).

[8]The social problem satisfies all the hypothesis of Theorem 1 in Makris (2001) with zero switching costs ($\Phi = 0$ in Makris' (2001) notation). The condition $\bar{H}_1(T^*) = \bar{H}_2(T^*)$ follows directly from equation [15] in Makris (2001: p.1939). See also Tomiyama (1985: Theorem 1).

[9] Parameter values are $\alpha = 2$, $m = n = G = 1$, $\gamma = 0.3$, $S_0 = 1000$, $\delta = 0.25$, $\psi = 0.06$, $\rho = 0.04$, $\sigma = 1$, $A_0 = 10$. Notice that, in the logarithmic case $\sigma = 1$, the indirect welfare function $V(T)$ can be computed in an easy manner because, as shown in footnote 5, the optimal consumption propensity in the resource-based economy equals $c_1^* = \rho/\psi$ in each $t \in [0, T)$ independently of the value of T. This implies that we have simple closed-form solutions for the conditional consumption path $\tilde{C}_1(t; T)$ for any value of T, and this allows us to obtain explicit expressions for the indirect welfare sub-functions $V_1(T)$ and $V_2(T)$.

[10]In the present model, the continuity of the general Hamiltonian $H(t) = H_1(t) + H_2(t)$ in instant T^* is guaranteed by the fulfillment of the optimality conditions (20) and (40) - that is, $\mu_1(T^*) = \mu_2(T^*)$ and $H_1(T^*) = H_2(T^*)$. See Tomiyama (1985) and Seierstad and Sydsaeter (1987) on this point.

[11]As shown in the Appendix - see equation (A26) - the optimal switching time is characterized by the condition $\ln(C_1(T^*)/C_2(T^*)) = \delta$.

[12]Substituting $\hat{C}_1 = \hat{c}_1 + \hat{Y}_1 = \hat{c}_1 + \hat{A} + \delta\hat{R} = \hat{c}_1 + \psi(1 - c_1) + \delta\hat{R}$ in (26), and plugging (27) in the resulting expression, we obtain (A13).

[13]In Lemma 4, the restrictions (33)-(34) are associated with an infinite switching time $T = \infty$. However, as shown in Lemma 6, if we set the switching instant equal to $T = T'$, the resource-based economy exhibits the same properties listed in Lemma 4: the optimal propensity to consume equals (30), output and consumption grow at the constant rate (31), knowledge grows at the constant rate (32) in each $t \in [0, T')$, and the optimal path is well-defined if and only if parameters satisfy (33)-(34).

[14]The result that c_1 coincides with optimal propensity c_1^* obtained for the case $T = \infty$ in Lemma 4 may suggest conjecturing that c_1^* equals $\bar{c}_1$ in each period independently of the value of switching time T. This

conjecture is wrong: it can be shown that, when the elasticity of intertemporal substitution differs from unity, $\sigma \neq 1$, we have $c_1^* = \bar{c}_1$ for $T = \infty$, $c_1^* = \bar{c}_1$ for $T = T^*$, but $c_1^* \neq \bar{c}_1$ for other finite values of $T \neq T^*$. The only case in which the optimal propensity in phase 1 is independent of the switching time T arises when preferences are logarithmic: as shown in footnote 5, setting $\sigma = 1$ implies $c_1^* = c_2^* = \rho/\psi$ independently of the switching time T.

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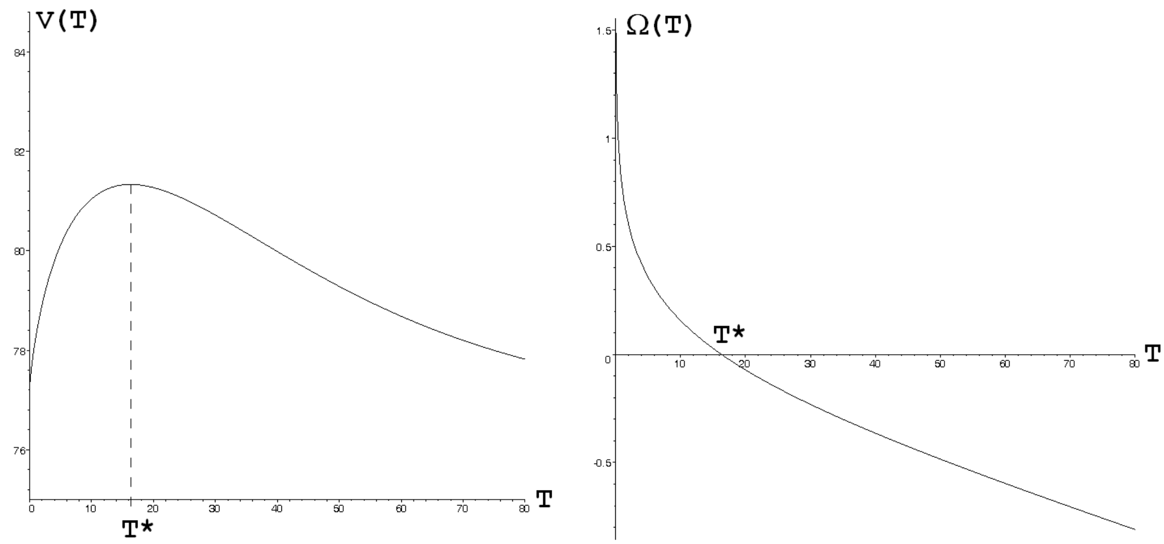

Figure 1: A numerical example of indirect welfare function and optimal switching time. The left graph depicts the welfare-timing relationship $V(T) = V_1(T) + V_2(T)$. The optimal switching time T^* corresponds to the horizontal intercept of the gap function $\Omega(T)$, depicted in the right graph.

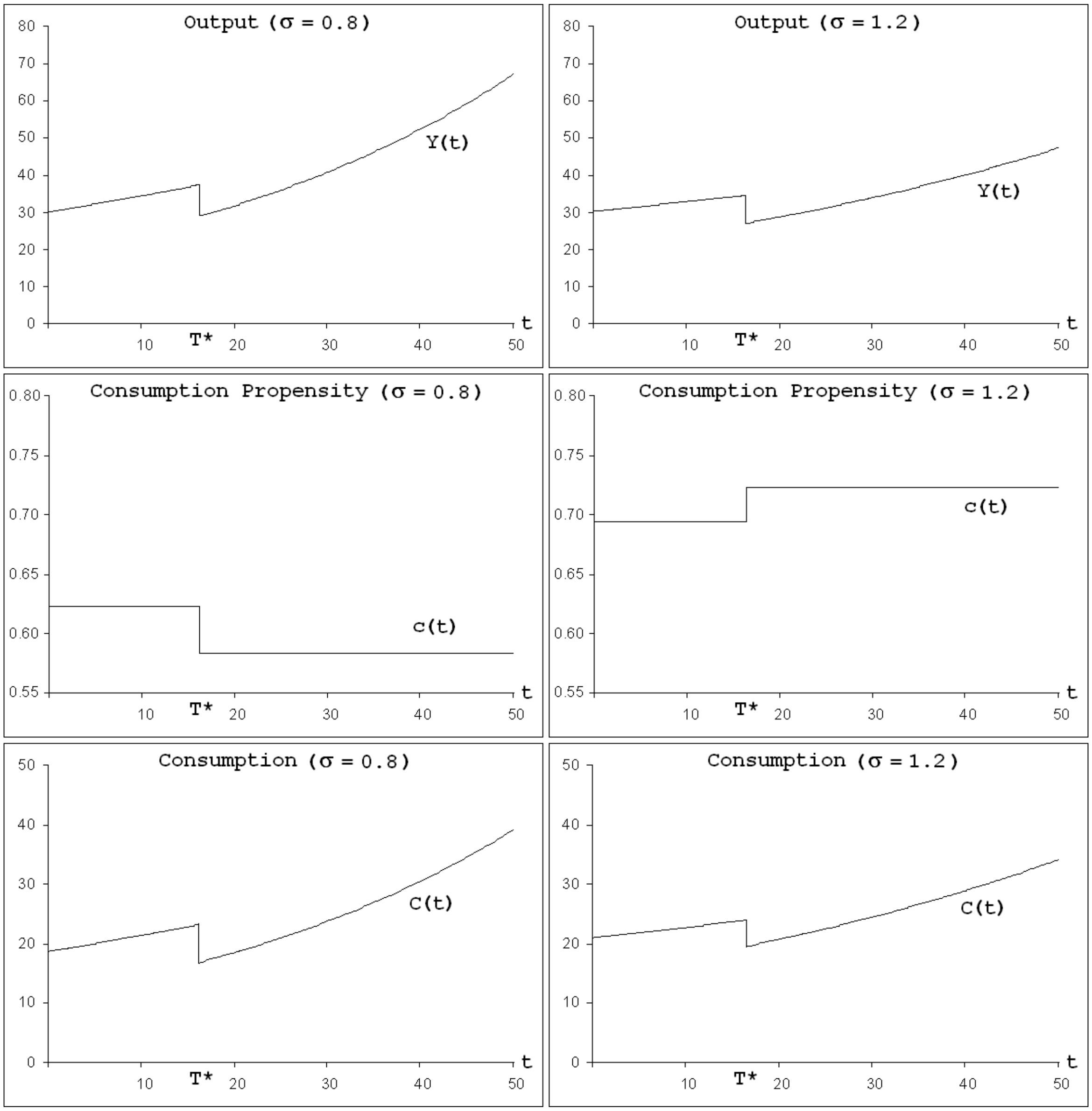

Figure 2: Optimal paths of output, consumption and consumption propensities for the two cases $\sigma = 0.8$ and $\sigma = 1.2$. See Table 1 for details, and footnote 9 for the list of parameter values.

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